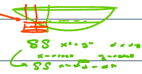
Math 53, Discussion 105

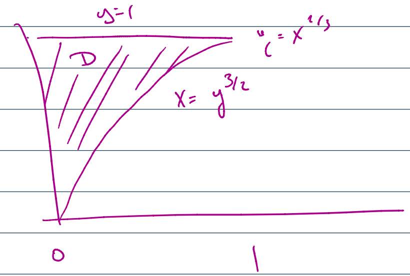
Monday, March 30, 2020 8:47 AM

1) Find the minimum and Maximum values of the function $f = (x-1)^2 + (y-1)^2$ on the unit disc $x^2 + y^2 \leq 1$
2) Calculate $\int_0^1 \int_{x^{\frac{2}{3}}}^1 x\cos(y^4)\,dy\,dx$
3) ~~Find the area of the region enclosed by the curve $x^2 + xy + y^2 = 1$~~

~~Hint: use the substitution $x = u + v\sqrt{3}, y = u - v\sqrt{3}$~~

4) Let R be the region above the cone $z = \sqrt{3x^2 + 3y^2}$ and below the sphere $z = \sqrt{9 - x^2 - y^2}$. Set up an integral or the volume of this region using the following integration orders:
   A) $dz\,dx\,dy$
   B) $dz\,dr\,d\theta$
   C) $dx\,dy\,dz$
5) A stick of length 1 is randomly dropped on a ruled sheet of paper, where the lines are spaced 2 units apart. What's the probability that the stick touches a line?



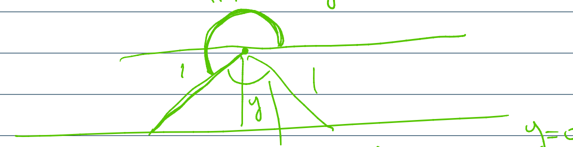


$D = \{(x,y)$ s.t. $0 \le y \le 1$ & $0 \le x \le y^{3/2}\}$

$I = \int_0^1 \int_0^{y^{3/2}} x\cos(y^4)\,dx\,dy$

Prob of hitting line

$\frac{1}{2}\int_{-1}^{1} \underline{\qquad\qquad}\, dy$

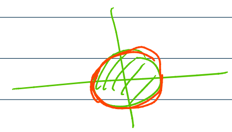


$\frac{1}{2\pi}$

all those θ's

$\boxed{\frac{1}{2\pi}\left(\pi - \sin^{-1}(y)\right)}$

Find the minimum and Maximum values of the function $f = (x-1)^2 + (y-1)^2$ on the unit disc $x^2 + y^2 \leq 1$



Extrem value theorem: every continuous fxn has a max & min on a closed region

1) look for max & min inside region

find pts s.t. $f_x = f_y = 0$

would get only at $(1,1)$ which isn't in region.

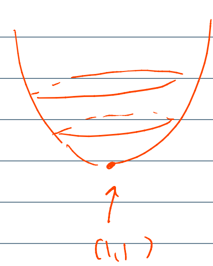
2) use Lagrange multiplier to find max & min on boundary

$g = x^2 + y^2$ equality!

$$\begin{cases} 2(x-1) = \lambda\, 2x \\ 2(y-1) = \lambda\, 2y \\ x^2 + y^2 = 1 \end{cases} \Rightarrow x = y$$

$\left(\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right)$ min

$\left(\frac{-1}{\sqrt{2}}, \frac{-1}{\sqrt{2}}\right)$ max



Calculate $\int_0^1 \int_{x^{\frac{2}{3}}}^1 x\cos(y^4)\,dy\,dx = I$

Switch order of integration

31.

Switch order of integration

$$I = \int_0^1 \int_0^{y^{3/2}} x\cos(y^4)\,dx\,dy$$

$$\frac{x^2}{2}\cos(y^4)\Big|_{x=0}^{x=y^{3/2}}$$

$$\int_0^1 \frac{y^3}{2}\cos(y^4)\,dy$$

$$\frac{1}{4}\,\frac{1}{2}\sin(y^4)\Big|_0^1 = \frac{1}{8}\sin(1)$$

1) Let R be the region above the cone $z = \sqrt{3x^2 + 3y^2}$ and below the sphere $z = \sqrt{9 - x^2 - y^2}$. Set up an integral or the volume of this region using the following integration orders:

A) $dz\,dx\,dy$

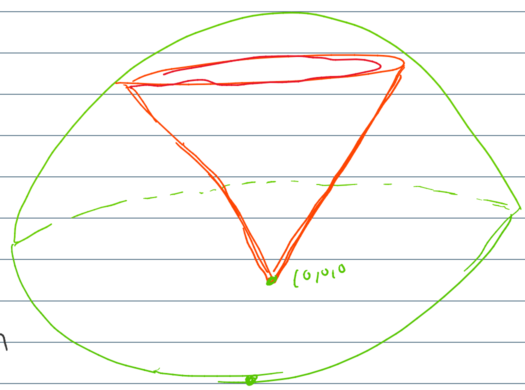
B) $dz\,dr\,d\theta$

C) $dx\,dy\,dz$

$z = 3$

$9 = 3x^2 + 3y^2$

$3 = x^2 + y^2$



a) $$\int_{-\frac{3}{2}}^{3/2} \int_{-\sqrt{\frac{9}{4} - y^2}}^{\sqrt{9/4 - y^2}} \int_{\sqrt{3x^2+3y^2}}^{\sqrt{9-x^2-y^2}} 1 \; dz\,dx\,dy$$

$$\sqrt{3x^2 + 3y^2} = \sqrt{9 - x^2 - y^2}$$